Inventory Control – Economic Order Quantity

LEARNING OBJECTIVES

1. Understand the importance of inventory management in general
2. Learn how to use the Economic Order Quantity (EOQ) model to manage inventory in situations where demand is constant over time
## CONTENTS

- Motivation
  - Economic Order Quantity Model
  - Extensions of Economic Order Quantity Model
  - Summary

### PRODUCTION AND OPERATIONS MANAGEMENT

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<th>Demand fulfillment</th>
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- Inventory management
- Fulfillment implementation
REASONS FOR HOLDING INVENTORY

• Customer service: Ensure high service level
• Demand uncertainty: Buffer against larger than expected demands
• Supply uncertainty: Delay and uncertainty in transportation
• Psychological impact: Consumer behaviors are affected by inventory
• Economies of scale: Allocate setup cost over a larger number of quantity
• Quality issues: Defective parts and workstation breakdowns
• Working-In-Process (WIP) inventory: Necessary for production

COST OF INVENTORY

• In US, the value of inventory is greater than $1,000,000,000,000 in total

• In many industries in US, the annual cost of inventory holding, shortage, and excess supply accounts for a significant portion of annual sales (e.g., 25% in the apparel industry)

• In a typical company, inventory cost accounts for
  - 30% of the current assets
  - 90% of the working capital
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MODELING ASSUMPTIONS

1. Production is instantaneous
2. No delivery leadtime
3. Demand is deterministic and known
4. Demand rate is constant over time
5. A production run incurs a fixed setup cost, regardless of production quantity
6. Consider the case of a single product
7. No backorders allowed
CONCEPTUAL FRAMEWORK

MODEL PARAMETERS AND DECISION VARIABLE

D  Demand rate (in units per year)
c  Unit production/purchase cost (in dollars per unit)
A  Fixed setup cost to produce/order one lot (in dollars)
h  Holding cost of one unit for a year (in dollars per unit per year)
Q  Lot size/order quantity (in units)  \(\leq\) decision variable

Comments:
h = ic, if the holding cost consists of only the interest on money tied up in inventory, where \(i\) is the effective annual interest rate

F = D/Q, production frequency, i.e., number of lots per year (times per year)
T = Q/D, cycle length of a lot (in years)
RELEVANT COSTS

1. Inventory holding cost per year =
   \[ \frac{Q}{2} \] is the average inventory level throughout the year

2. Setup cost per year =
   \[ F = \frac{D}{Q} \] is number of productions per year

3. Production cost per year =
   which is independent of the decision variable Q

So, total cost per year is \( Y(Q) = \)
ECONOMIC ORDER QUANTITY

First order condition

Second order condition

Total cost at optimality

EXAMPLE

Data
• 20 units of demand per week
• $4 production price for one unit
• $20 for each setup
• 25% annual interest rate for on money tied up in inventory

Solution
SENSITIVITY ANALYSIS

Sensitivity of total cost $Y(Q)$ with respect to lot size $Q$ (ignoring production cost)

SENSITIVITY ANALYSIS: A SECOND VIEW

Instead of lot size $Q$, take cycle length $T$ as the decision variable
POWERS-OF-2 POLICY: IDEA

- The powers-of-2 policy means that we produce quantity at intervals given by powers of 2
- This policy is, of course, a heuristic approach
- This is easy to use and can coordinate shipments of different products

<table>
<thead>
<tr>
<th>Interval</th>
<th>Week</th>
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<tr>
<td>0</td>
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<tr>
<td>1 = 2^0</td>
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<tr>
<td>2 = 2^1</td>
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<td>4 = 2^2</td>
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<tr>
<td>8 = 2^3</td>
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POWERS-OF-2 POLICY: IMPLEMENTATION

Assume that
1) the optimal interval obtained from the EOQ model is $T^*$
2) $T^*$ is between $2^m$ and $2^{m+1}$
3) we use powers-of-2 policy

Thus, produce at a cycle length $T = 2^m$ if $2^m \leq T^* \leq 2^m \sqrt{2}$
produce at a cycle length $T = 2^{m+1}$ if $2^m \sqrt{2} \leq T^* \leq 2^{m+1}$

The error of using the above policy can at most result in an error of 6% with regard to the inventory holding cost plus the setup cost
EXAMPLE CONTINUOUS

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ECONOMIC PRODUCTION LOT (1)

Assumption
Instead of being instantaneous, the production takes time

Conceptual framework
Inventory level

ECONOMIC PRODUCTION LOT (2)

Additional parameter
P  Production rate (in units per year)

Analysis
Lot size Q is still the decision and cost needs to be modified
The uptime (when production happens) T₁ =
The downtime (when production does not happen) T₂ =
Maximum inventory level in a cycle =
Average inventory level in a cycle =

Total cost per year = \( \frac{hQ}{2} \left( 1 - \frac{D}{P} \right) \) + \( \frac{AD}{Q} \) + cD

Optimal lot size = \( Q^* = \frac{2AD}{h(1 - \frac{D}{P})} \)
ECONOMIC PRODUCTION LOT (3) - EXAMPLE

Data
• 20 units of demand per week
• $4 production price for one unit
• $20 for each setup
• 25% annual interest rate for on money tied up in inventory
• 100 units of production per week

Solution

EOQ WITH ALL-UNIT DISCOUNTS (1)

Assumption
Instead of being constant, the unit cost c now depends on the lot size Q according to an all-unit discount schedule c(Q)
EOQ WITH ALL-UNIT DISCOUNTS (2)

Total cost function
\[ Y(Q) = \left[ c(Q) \right] \frac{Q}{2} + A \frac{D}{Q} + c(Q)D \]
where
\[ c(Q) = \begin{cases} c_0 & \text{if } 0 \leq Q < Q_1 \\ c_1 & \text{if } Q_1 \leq Q < Q_2 \\ c_2 & \text{......} \end{cases} \]

EOQ WITH ALL-UNIT DISCOUNTS (3)

Idea: two-phase optimization

Phase 1: For each unit cost \( c_j \), find the minimum cost within its range

Phase 2: Find the lowest among all those minimum costs

Algorithm
- Let \( Q^*_j = \sqrt{\frac{2AD}{c_j}} \) be the value obtained from EOQ using unit cost \( c_j \)
- If \( Q^*_j \) is in quantity range for \( c_j \) (i.e., \( Q_j \leq Q^*_j < Q_{j+1} \)), set \( Q^*_j = Q^*_j \);
  Otherwise, pick for \( Q^*_j \) the boundary point of the quantity range which is closest to \( Q^*_j \)
- Use \( Q^*_j \) to calculate \( Y^*_j \)
- Select among all \( j \) to identify the optimal \( Q^* \) and \( Y^* \)
EOQ WITH ALL-UNIT DISCOUNTS (4) - EXAMPLE

Data
D = 600 units per year
A = $8 per order
i = 20% per year
c(Q): c_0 = $0.30, c_1 = $0.29, c_2 = $0.28

Case 1: Q_1 = 500, Q_2 = 1,000
j = 0:
j = 1:
j = 2:

Case 2: Q_1 = 300, Q_2 = 400
j = 0:
j = 1:
j = 2:

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SUMMARY

• The EOQ model is the most fundamental inventory-control model
• The EOQ model can be applicable wherever demand is constant over time
• The basic tradeoff underlying the EOQ model is between the inventory holding cost and the setup cost
• The EOQ model is rather robust in the sense that the total cost is relatively insensitive to errors on the lot size decision $Q$

ANNOUNCEMENTS

• For the EOQ model, read Section 2.1-2.2