8.21 A builder wishes to use a 5-m beam to span two walls that must support the loads given in Figure P8.21. The building code states that the maximum bending moment for a beam of this construction (cross section and material) is 13 kN · m. Compute the reaction forces, shear, and moment functions, and plot the shear and moment functions. Does the beam satisfy the code?

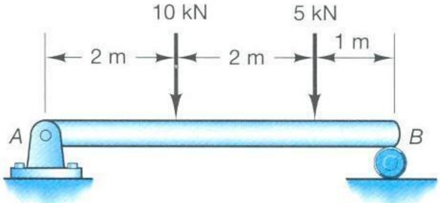


Figure P8.21

8.22 Compute the reaction forces and the shear and moments along the beam for the given load shown in Figure P8.22. What is the maximum value of the bending moment, and where in the beam does it occur?

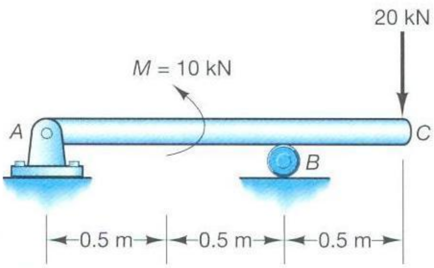


Figure P8.22

8.25 Determine the reaction forces and the internal forces and moments for the system illustrated in Figure P8.25. Plot the shear and bending moment functions against the distance x.

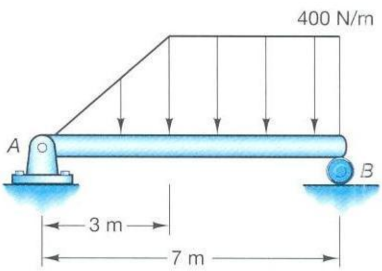


Figure P8.25

8.28 A cantilever beam is used as a crude model of a building. (See Figure P8.28.) Compute the reaction forces and the internal shear force and bending moment for a strong wind load modeled as $w(x) = (x^3 - 100x^2)\,10^{-4}$ lb/ft.

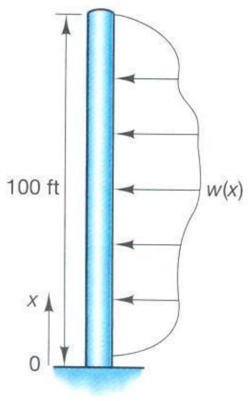


Figure P8.28

8.34 In Figure P8.34, compute the reaction forces, shear force, and bending moment for the combination of point load P and distributed load w_0, in terms of the constants a, b, L, P, and w_0 as a function of x. Assume that $a < b$.

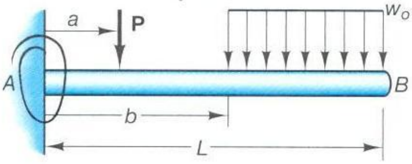


Figure P8.34

8.35 In Figure P8.35, compute the reaction forces, the shear force, and the bending moment function at each point x between the values of 0 and L, due to the applied distributed load and the applied moment M in terms of the constants a, b, L, w_0, and M. Assume that $a < b$.

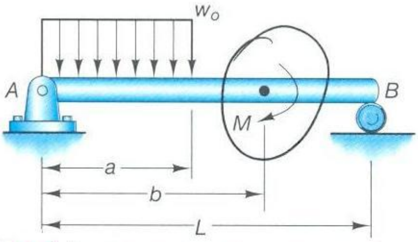


Figure P8.35

8.36 In Figure P8.36, compute the reaction forces, the shear force, and the bending moment due to the two applied point forces P and the distributed load w_0, in terms of the constants L, P, and w_0, for each value of $x = 0 < x < 6L$.

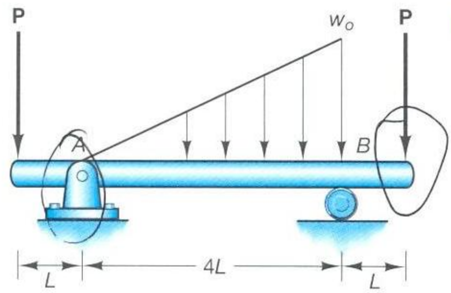


Figure P8.36

8.41 In Figure P8.41, compute the reaction forces, the shear force, and the bending moment due to the distributed load of intensity w_0, in terms of the constants L and w_0, for each value of x between 0 and L.

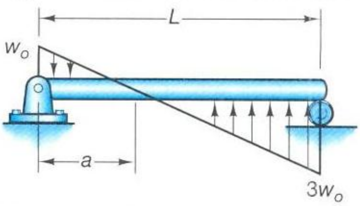


Figure P8.41

8.21 From S8.21a:

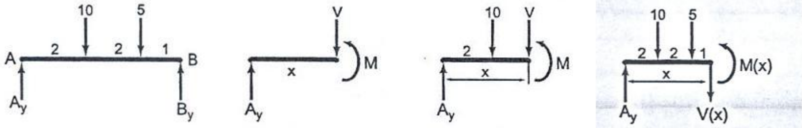


FIGURE S8.21a FIGURE S8.21b FIGURE S8.21c FIGURE S8.21d

$$\sum F_y = 0: \quad A_y + B_y - 10 - 5 = 0, \sum M_A = 0: \quad -20 - 20 + 5B_y = 0$$

$$\underline{B_y = 8 \text{ kN}}$$

and

$$\underline{A_y = 7 \text{ kN}.}$$

From S8.21b:

$$\sum F_y = 0: \quad 7 - V = 0$$

or

$$\underline{V(x) = 7 \text{ kN } 0 \le x < 2} \text{ (down)}$$

and

$$\sum M_A = 0 \quad -7x + M = 0$$

or

$$\underline{M(x) = 7x \text{ kN} \cdot \text{m}, \; 0 \le x < 2.}$$

From S8.21c:

$$\sum F_y = 0$$

yields

$$7 - 10 - V = 0$$

or

$$\underline{V(x) = -3 \text{ kN}, \; 2 < x < 4} \text{ (up)}$$

$$\sum M_A = 0: \quad -(x)(-3) + M(x) - 20 = 0$$

or

$$\underline{M(x) = 20 - 3x \text{ kNm}, \; 2 < x < 4}$$

From S8.21d:

$$\sum F_y = 0: \quad 7 - 15 - V(x)$$

or

$$\underline{V(x) = -8 \text{ kN}, \; 4 < x < 5} \text{ (up)}$$

and

$$\sum M_A = 0: \quad (-2)(10) - (4)(5) - x(-8) + M(x) = 0$$

or

$$\underline{M(x) = 40 - 8x \quad 4 < x \le 5}$$

The plots of $V(x)$ and $M(x)$ are

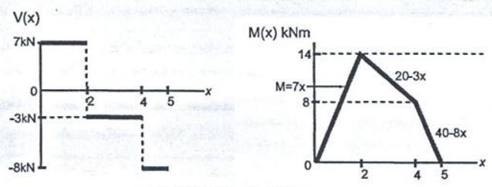


FIGURE S8.21e

The maximum value of the bending moment is <u>14 kNm</u>, so the beam will <u>not</u> hold the desired load and does <u>not</u> satisfy the code.

8.22 From S8.22a: $\sum F_y = A_y + B_y - 20 = 0$

$$\sum M_A = 10 \text{ kN} - (1.5)(20) + (1)(B_y) = 0$$

so that

$$\underline{B_y = 20 \text{ kN}},$$

$$\underline{A_y = 0}$$

From S8.22b:

$$\sum F_y = 0: \quad A_y + V(x) = 0$$

$$\underline{V(x) = 0: \quad 0 \le x < 0.5 \text{ m}}$$

$$\sum M_A = 0: \quad M(x) + xV(x) = 0,$$

$$\underline{M(x) = 0: \quad 0 \le x < 0.5 \text{ m}}$$

From S8.22c:

$$\sum F_y = 0: -A_y + V(x) = 0$$

$$\underline{V(x) = 0 \quad 0.5 < x < 1.0 \text{ m}}$$

$$\sum M_A = 0: 10 + M(x) = 0$$

$$\underline{M(x) = -10\text{Nm} \quad 0.5 < x < 1.0 \text{ m}}$$

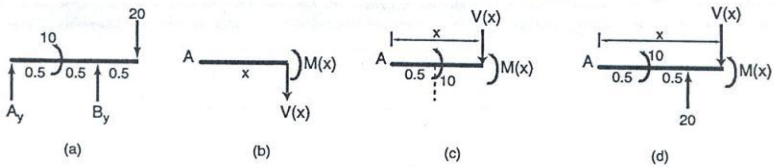


FIGURE S8.22a

From S8.22d:

$$\sum F_y = 0: 20 - V(x) = 0$$

$$\underline{V(x) = 20 \text{ kN} \quad 1.0 < x < 1.5 \text{ m}}$$

$$\sum M_A = 0: \quad 10 + 20 - 20x + M(x) = 0$$

$$\underline{M(x) = 20x - 30 \quad 1.0 < x < 1.5 \text{ m}}$$

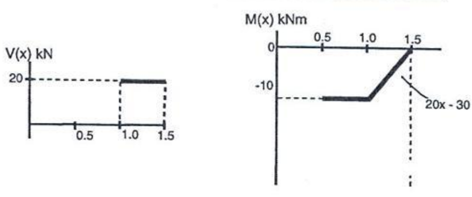


FIGURE S8.22b

8.25 The equivalent point force and centroid are used here to construct a free-body diagram of the whole structure.

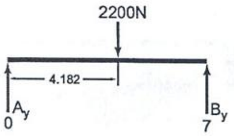


FIGURE S8.25a

Summing forces:

$$A_y + B_y = 2200$$
$$\sum M_A : -(2200)(4.182) + 7B_y = 0$$

so that

$$\underline{B_y = 1314.3 \text{ N}}$$

and

$$A_y = 885.7 \text{ N}$$

Next make a free body diagram of the section to the left of a cut at x before $x = 3$ and again at x after $x = 3$. The height of load is given by similar triangles

$$\frac{h}{x} = \frac{400}{3}$$
$$h = \frac{400x}{3}$$
$$P = \frac{200}{3}x^2$$

$$\sum F = 0 : \quad 885.7 - V(x) - \frac{200}{3}x^2 = 0$$
$$\underline{V(x) = 885.7 - 66.67x^2 \text{ N}, \ 0 \le x < 3 \text{ m}}$$
$$\sum M_x = 0 : \quad M(x) - 885.7x - \frac{x}{3}(\frac{200}{3}x^2)$$
$$\underline{M(x) = 885.7x - 22.22x^3, \ \ 0 \le x < 3}$$

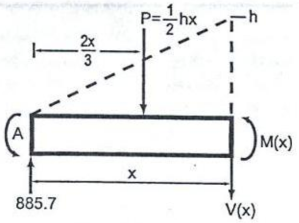


FIGURE S8.25b

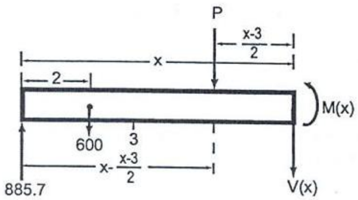


FIGURE S8.25c

From S8.25c:

The cut at $x > 3$ yields the force P is

$$(x - 3)(400) \text{ N}$$
$$\sum F_y = 0 : \quad 885.7 - 600 - 400x + 1200 - V(x) = 0$$

so that

$$\underline{V(x) = -400x + 1485.7 \text{ N}, \quad 3 < x \le 7}$$

and

$$\sum M_A = -1200 - (\frac{x}{2} + \frac{3}{2})(400x - 1200) - x(-400x + 1485.7) + M(x) = 0$$

so

$$\underline{M(x) = -200x^2 + 1485.7x - 600 \text{ Nm}, \ 3 < x \le 7}$$

8.28 The equivalent point load and position are:

$$W = 10^{-4} \int_0^{100} (x^3 - 100x^2)dx = -8.33 \times 10^2 \text{ lb}$$

$$x_c = \frac{10^{-4}}{W} \int_0^{100} x(x^3 - 100x^2)dx = 60.0 \text{ ft}$$

This yields the free-body diagram used for calculating the reactions at the base

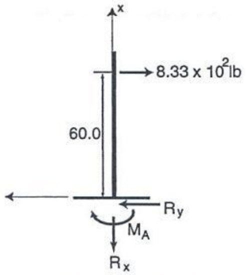


FIGURE S8.28a

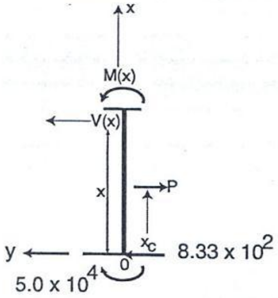


FIGURE S8.28b

$$\sum F_x = 0 \rightarrow R_x = 0$$

$$\sum Fy = R_y - 8.33 \times 10^2 = 0$$

or

$$\underline{R_y = 8.33 \times 10^2 \text{ lb}}$$

$$\sum M_0 = (60)(8.33 \times 10^2) + M_A = 0$$

$$\underline{M_A = -5.0 \times 10^4 \text{ lb} \cdot \text{ft}}$$

The load acting on the cut section from 0 to x is

$$P = 10^{-4} \int_0^x (x^3 - 100x^2)dx = \frac{x^3(3x - 400)}{120000}$$

acting at

$$x_c = \frac{1}{P} \int_0^x x(x^3 - 100x^2)dx = \frac{12}{5} \times \frac{x(x - 125)}{(3x - 400)}$$

From the free body diagram (S8.28b)

$$\sum F_y = 0: \quad 8.33 \times 10^2 - \frac{x^3(3x - 400)}{120000} - V(x) = 0$$

or

$$\underline{V(x) = \frac{-x^3(3x - 400)}{120000} + 8.33 \times 10^2 \text{ lb}}$$

$$\sum M_x = 0: \quad M(x) - 5.0 \times 10^4 - (8.33 \times 10^2)x$$

$$+ \left(x - \frac{12}{5}\frac{x(x - 125)}{(3x - 400)}\right) \frac{x^3(3x - 400)}{120000} = 0$$

$$\underline{M(x) = 5.0 \times 10^4 + (8.33 \times 10^2)x - \frac{x^4}{60}10^{-4}(3x - 500) \text{ lbft}}$$

Note that

$$\frac{dM(x)}{dx} = V(x)$$

as it should.

8.34 Reactions at the support:

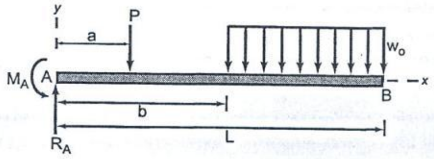


FIGURE S8.34a

$$\sum F_y = 0 \Rightarrow -w_0(L-b) - P + R_A = 0$$

$$\Rightarrow R_A = P + w_0(L-b) \uparrow$$

$$\sum M_B = 0 \Rightarrow +\frac{w_0}{2}(L-b)^2 + P(L-a) - R_A(L) + M_A = 0$$

$$\Rightarrow M_A = PL + w_0L(L-b) - P(L-a) - \frac{w_0}{2}(L-b)^2$$

$$M_A = Pa + w_0(L-b)\{L - \frac{(L-b)}{2}\} = Pa + w_0(L-b)\{(L+b)/2\}$$

$$M_A = Pa + \frac{w_0}{2}(L^2 - b^2)$$

Loading changes intervals:

$$0 \le x < a, \quad a \le x < b, \quad b \le x \le L.$$

To find the internal loading (forces/moments), the following cuts must be made:

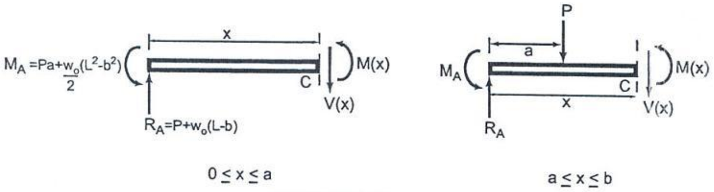


FIGURE S8.34b

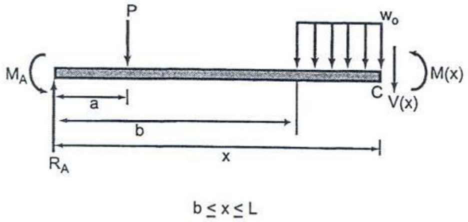


FIGURE S8.34c

Equilibrium equations:

$$0 \le x < a \Rightarrow \sum F_y = 0 \Rightarrow \underline{V(x): \quad P + w_0(L-b)}$$

$$\sum M_c = 0 \Rightarrow M(x) + (Pa + \frac{w_0}{2}(L^2 - b^2)) - (P + w_0(L-b))x = 0.0$$

$$\underline{M(x) = (P + w_0(L-b))x - (Pa + \frac{w_0}{2}(L^2 - b^2))}$$

$$a \le x < b \Rightarrow \sum F_y = 0 \Rightarrow \underline{V(x) = w_0(L-b)}$$

$$\sum M_c = 0 \Rightarrow M(x) + Pa + \frac{w_0}{2}(L^2 - b^2) - (P + w_0(L-b))x + P(x-a) = 0$$

$$\underline{M(x) = w_0(L-b)x - \frac{w_0}{2}(L^2 - b^2)}$$

$$b \le x < L \Rightarrow \sum F_y = 0 \Rightarrow -w_0(x-b) - P + (P + w_0(L-b)) - V(x) = 0$$

$$\Rightarrow \underline{V(x) = -w_0(x-b) + w_0(L-b) = w_0(L-x)}$$

$$\sum M_c = 0 \Rightarrow +\frac{w_0}{2}(x-b)^2 + P(x-a) + [Pa + \frac{w_0}{2}(L^2 - b^2)] - (P + w_0(L-b))x + M(x) = 0$$

$$\Rightarrow \underline{M(x) = -\frac{w_0}{2}(x-b)^2 + w_0(L-b)x - \frac{w_0}{2}(L^2 - b^2) = -\frac{w_0}{2}(L-x)^2}$$

8.35 Compute the reaction forces from a free-body diagram of the entire structure:

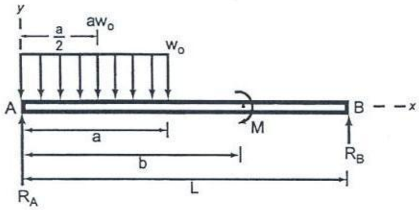


FIGURE S8.35a

$$\sum F_y = 0 \Rightarrow +(a)w_0 = R_A + R_B$$

$$\sum M_A = 0 \Rightarrow -M + LR_B - \frac{a^2}{2}w_0 = 0$$

$$\Rightarrow R_B = \frac{a^2}{2L}w_0 + \frac{M}{L} \uparrow$$

$$R_A = w_0 a(1 - \frac{a}{2L}) - \frac{M}{L} \uparrow$$

The loading changes in the intervals:

$$0 \le x < a \quad a \le x < b \quad b \le x < L.$$

To find the internal loading (force/moment) in every segment, the cuts shown in S8.35a, S8.35b and S8.35c must be made. Equilibrium equations for each segment become:

$$0 \le x < a \quad \sum F_y = 0 \Rightarrow \underline{V(x) = w_0 a(1 - \frac{a}{2L}) - \frac{M}{L} - w_0 x}$$

$$\sum M_c = 0 \Rightarrow +w_0\frac{x^2}{2} - R_A x + M(x) = 0$$

$$\underline{M(x) = (w_0 a(1 - \frac{a}{2L}) - \frac{M}{L})x - \frac{w_0}{2}x^2}$$

$$a \le x < b \quad \sum F_y = 0 \Rightarrow \underline{V(x) = -\frac{a^2}{2L}w_0 - \frac{M}{L}}$$

$$\sum M_c = 0 \Rightarrow aw_0(x - \frac{a}{2}) - R_A x + M(x) = 0$$

$$\underline{M(x) = -\frac{a^2}{2L}w_0 x - \frac{M}{L}x + \frac{a^2 w_0}{2}}$$

$$b \le x \le L \quad \sum F_y = 0 \Rightarrow V(x) = -\frac{a^2}{2L}w_0 - \frac{M}{L}$$

$$\sum M_c = 0 \Rightarrow M(x) - M - R_A x + w_0 a(x - \frac{a}{2}) = 0$$

$$\underline{M(x) = -\frac{a^2}{2L}w_0 x + M(1 - \frac{x}{L}) + \frac{a^2 w_0}{2}}$$

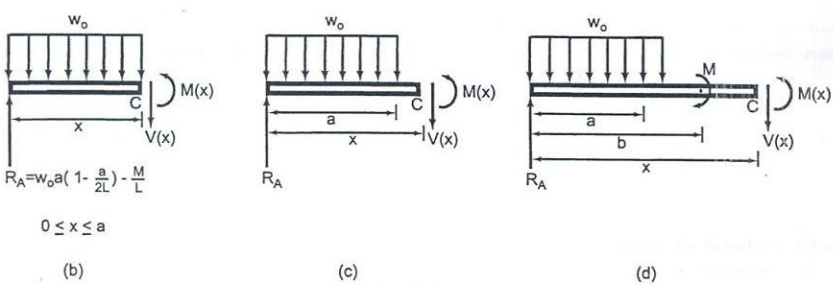


FIGURE S8.35b

8.36 Compute the support reactions from the free-body diagram with the distributed load modeled as a point load:

$$\sum F_y = 0 \Rightarrow R_A + R_B = 2P + 2Lw_0$$

$$\sum M_A = 0 \Rightarrow -5LP - (2Lw_0)(\frac{2}{3}(4L)) + PL + 4LR_B = 0$$

$$\Rightarrow R_A = P + \tfrac{4}{3}w_0L$$

$$R_A = P + \tfrac{2}{3}w_0L$$

The loading changes in the intervals:

$$0 \leq x < L \quad L \leq x \leq 4L \quad 5L \leq x \leq 6L$$

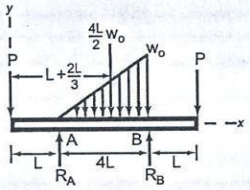


FIGURE S8.36a

To find the internal forces/moments draw the FBD after making the cuts illustrated.

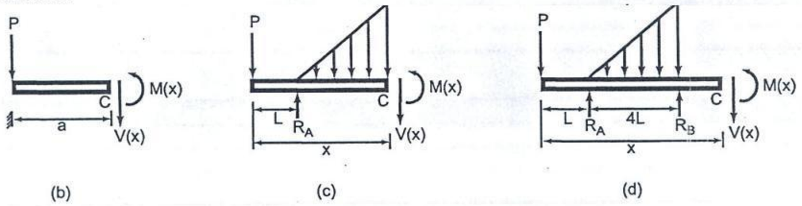


FIGURE S8.36b,c,d

$0 \leq x < L$

$\sum F_y = 0 \Rightarrow \underline{V(x) = -P}$

$\sum M_c = 0 \Rightarrow \underline{M(x) = -Px}$

$\underline{L \leq x < 5L}$

At this stage we need to know the value of the load at x (i.e. w). Consider the total distributed load: from similar triangles: $\frac{w}{x-L} = \frac{w_0}{4L}$ so that $w = \frac{w_0(x-L)}{4L}$

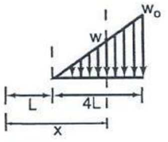


FIGURE S8.36e

consider the free body diagram $\Rightarrow$ (Fig. S8.36c)

$\sum F_y = 0 \Rightarrow -P + R_A - V(x) - \frac{1}{2}(x-L)(w_0\frac{(x-L)}{4L}) = 0$

$\Rightarrow V(x) = -\frac{1}{8L}w_0(x-L)^2 - P + R_A$

$V(x) - \frac{-1}{8L}w_0(x-L)^2 + \frac{2}{3}w_0L$

$\sum M_c = 0 \Rightarrow M(x) - R_A(x-L) + Px + \frac{1}{3}(x-L)\cdot\left(\frac{w_0(x-L)}{4L}\right)\left(\frac{1}{2}(x-L)\right) = 0$

$M(x) = -Px - \frac{1}{6}(x-L)^3\frac{w_0}{4L} + \left(P + \frac{2}{3}w_0L\right)(x-L)$

$\Rightarrow \underline{M(x) = -\frac{1}{24}\frac{w_0}{L}(x-L)^3 + \frac{2}{3}w_0L(x-L) - PL}$

$\underline{5L \leq x < L}$ (Fig. S8.36d)

$\sum F_y = 0 \Rightarrow -V(x) - P + R_A + R_B - w_0\frac{(4L)}{2} = 0$

$V(x) = -P + 2P + 2Lw_0 - 2Lw_0$

$\underline{V(x) = P}$

$\sum M_c = 0 \Rightarrow -R_B(x-5L) - R_A(x-L) + w_0(2L)(x-(\frac{2}{3}(4L)+L)) + Px + M(x) = 0$

$\Rightarrow \underline{M(x) = -P(6L-x)}$

8.41 The reactions at the supports are found from S.41a and similar triangles:

$w_0/a = 3w_0/(L-a) \Rightarrow \frac{(L-a)}{3w_0} = \frac{a}{w_0}$ so that $L - a = 3a \Rightarrow L = 4a \Rightarrow a = L/4$

$\sum F_y = 0 \Rightarrow R_A + R_B - \frac{1}{2}aw_0 + \frac{1}{2}(L-a)(3w_0) = 0$

$\Rightarrow R_A + R_B = \frac{1}{2}\left(\frac{L}{4}\right)w_0 - \frac{1}{2}\left(L - \frac{L}{4}\right)3w_0$

$$R_A + R_B = -Lw_0 \qquad (1)$$

$\sum M_B = 0 \Rightarrow$

$$-LR_A - \left(\frac{3}{2}w_0\right)\left(\frac{3L}{4}\right)\left(\frac{3L}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{w_0}{2}\right)\left(\frac{L}{4}\right)\left(\frac{2}{3}\left(\frac{L}{4}\right) + \frac{3L}{4}\right) = 0 \qquad (2)$$

Solving yields: $R_A = -\frac{w_0 L}{6}$ and $R_B = -\frac{5}{6}w_o L$

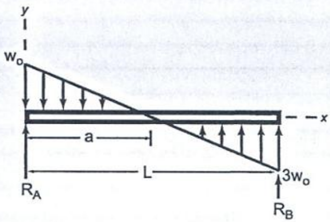
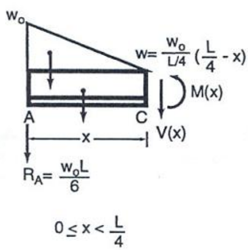


FIGURE S8.41a

To find the internal force/moment at each segment make the cuts illustrated:



(b)

FIGURE S8.41b

$0 \leq x < L/2$ $\sum F_y = 0$ $-V(x) - R_A - x\left(\frac{w_o}{L/4}\right)\left(\frac{2}{4} - x\right) - \frac{1}{2}x\left(w_0 - \frac{w_o}{L/4}\left(\frac{L}{4} - x\right)\right) = 0$

$\underline{V(x) = -w_0 x\left(1 - \frac{2x}{L}\right) - \frac{w_0 L}{6}}$

$\sum M_C = 0 : \frac{w_o L}{6}x + \frac{x^2}{2}\left(\frac{w_0}{(L/4)}\left(\frac{L}{4} - x\right)\right) + \frac{2}{3}x\left(w_0 - \frac{2_0}{(L/4)}\left(\frac{L}{4} - x\right)\right)\frac{1}{2}x + M(x) = 0$

or $\underline{M(x) = -\frac{w_0 x^2}{2}\left(1 - \frac{4x}{3L}\right) - \frac{w_0 L}{6}x}$

$L/4 \leq x < L \Rightarrow$ same solution, since $w(x)$ is the same.