2.54 First form the general expressions for the tension in each of the three cables:

$$\mathbf{T_A} = \frac{T_A}{\sqrt{118}}(3\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}) = T_A(.276\mathbf{i} - .276\mathbf{j} + .921\mathbf{k})$$

$$\mathbf{T_B} = \frac{T_B}{\sqrt{125}}(-5\mathbf{i} + 10\mathbf{k}) = T_B(-.447\mathbf{i} + .894\mathbf{k})$$

$$\mathbf{T_C} = \frac{T_C}{\sqrt{136}}(6\mathbf{j} + 10\mathbf{k}) = T_C(.514\mathbf{j} + .857\mathbf{k}).$$

Then write the vector equilibrium equation:

$$\mathbf{T_A} + \mathbf{T_B} + \mathbf{T_C} = \mathbf{R} = 250\mathbf{k}\ \mathrm{N}$$

which gives 3 scalar equations:

$$.276T_A - .447T_B = 0$$

$$-.276T_A + .514T_C = 0$$

$$.921T_A + .894T_B + .587T_C = 250\ \mathrm{N}$$

which can be written in matrix form as

$$\begin{bmatrix} .276 & -.447 & 0 \\ -.276 & 0 & .514 \\ .921 & .894 & .587 \end{bmatrix} \begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 250 \end{pmatrix}$$

and solved using computational software

$$\begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 129.32 \\ 79.85 \\ 69.44 \end{pmatrix}\ \mathrm{N}.$$

■

2.118 Define the vector:

$$\mathbf{D} = 191\mathbf{i} + 217\mathbf{j} - 68\mathbf{k}.$$

The magnitudes can be computed from the following equations:

$$A = \frac{\mathbf{D} \cdot (\mathbf{b} \times \mathbf{c})}{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})} = 309.476$$

$$B = \frac{\mathbf{D} \cdot (\mathbf{c} \times \mathbf{a})}{\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})} = 7.517$$

$$C = \frac{\mathbf{D} \cdot (\mathbf{a} \times \mathbf{b})}{\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})} = -79.431.$$

The scalar equations written in matrix form are:

$$\begin{bmatrix} 0.6 & 0.707 & 0 \\ 0.8 & 0 & 0.385 \\ 0 & 0.707 & 0.923 \end{bmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 191 \\ 217 \\ -68 \end{pmatrix}.$$

This system can be solved using computational software:

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 309.476 \\ 7.517 \\ -79.431 \end{pmatrix}.$$

2.67 First, define all unit vectors:

$$\mathbf{p} = (\cos(30°)\sin(40°))\mathbf{i} + (\cos(30°)\cos(40°))\mathbf{j} + \sin(30°)\mathbf{k} = 0.557\mathbf{i} + 0.663\mathbf{j} + 0.5\mathbf{k}$$

$$\mathbf{a} = \mathbf{i}$$

$$\mathbf{b} = \cos(30°)\mathbf{i} + \sin(30°)\mathbf{j} = 0.866\mathbf{i} + 0.5\mathbf{j}$$

$$\mathbf{c} = c_x\mathbf{i} + c_y\mathbf{j} + c_z\mathbf{k}$$

and the force vectors are:

$$\mathbf{P} = 500\mathbf{p} \text{ N}$$

$$\mathbf{A} = A\mathbf{a} \text{ N}$$

$$\mathbf{B} = 300\mathbf{b} \text{ N}$$

$$\mathbf{C} = 400\mathbf{c} \text{ N}$$

The 4 equations in 4 unknowns A, c_x, c_y, c_z can be written by equating **i**, **j** and **k** components of the vector equation $\mathbf{P} = \mathbf{A} + \mathbf{B} + \mathbf{C}$ and noting that the sum of the squares of the direction cosines of **c** must equal unity:

$$c_x^2 + c_y^2 + c_z^2 = 1$$

$$A + 259.81 + 400c_x = 278.34$$

$$150.0 + 400c_y = 331.71$$

$$400c_z = 250.$$

These can be solved using computational software:

$$A = 272.46 \text{ N}$$

$$c_x = -0.635$$

$$c_y = 0.454$$

$$c_z = 0.625.$$

(Note that the following is also a solution.)

$$A = -235.41 \text{ N}$$

$$c_x = 0.635$$

$$c_y = 0.454$$

$$c_z = 0.625.$$

3.16 From the free body diagram of the connection at A the equilibrium equation is

$$T_1\hat{e}_1 + T_2\hat{e}_2 + T_3\hat{e}_3 + 5000\hat{e}_f = 0$$

which can be written as the matrix vector equation

$$[\hat{e}_1 \vdots \hat{e}_2 \vdots \hat{e}_3] \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = -5000\hat{e}_f$$

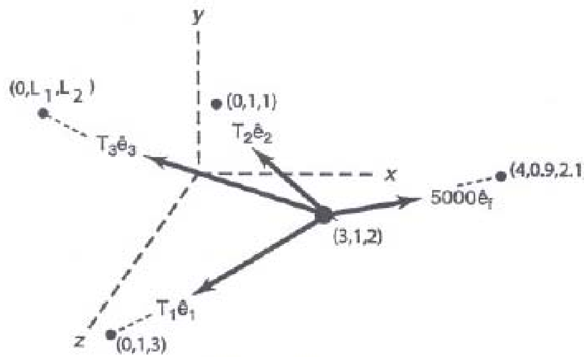

FIGURE S3.16

To calculate the unit vector along each rope by using the position vector of each rope, note that ($\ell_1 = 2$, $\ell_2 = 2$)

$$\mathbf{r}_1 = (0-3)\hat{\mathbf{i}} + (1-1)\hat{\mathbf{j}} + (3-2)\hat{\mathbf{k}}$$

$$\hat{e}_1 = \mathbf{r}_1/|\mathbf{r}_1| = -.949\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0.316\hat{\mathbf{k}}$$

$$\mathbf{r}_2 = (0-3)\hat{\mathbf{i}} + (1-1)\hat{\mathbf{j}} + (1-2)\hat{\mathbf{k}}$$

$$\hat{\mathbf{e}}_2 = \mathbf{r}_2/|\mathbf{r}_2| = -.949\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - 0.316\hat{\mathbf{k}}$$

$$\mathbf{r}_3 = (0-3)\hat{\mathbf{i}} + (\ell_1 - 1)\hat{\mathbf{j}} + (\ell_2 - 2)\hat{\mathbf{k}}$$

$$\hat{\mathbf{e}}_3 = (\mathbf{r}_3/\mathbf{r}_3) = -0.949\hat{\mathbf{i}} + 0.316\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$$

$$\mathbf{r}_f = (4-3)\hat{\mathbf{i}} + (.9-1)\hat{\mathbf{j}} + (2.1-2)\hat{\mathbf{k}}$$

$$\hat{\mathbf{e}}_f = \mathbf{r}_f/|\mathbf{r}_f| = .99\hat{\mathbf{i}} - .099\hat{\mathbf{j}} + .099\hat{\mathbf{k}}$$

Substitution of the unit vector values into the matrix equation yields

$$\begin{bmatrix} -.949 & -.949 & -.949 \\ 0 & 0 & .316 \\ .316 & -.316 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = - \begin{bmatrix} 4950.74 \\ -495.07 \\ 495.07 \end{bmatrix}$$

which can be solved by matrix inversion to yield ($L1$ etc. used to denote ℓ.)

$$T_1 = 1043.71 \text{ N} \quad T_2 = 2609.27 \text{ N} \quad T_3 = 1565.56 \text{ N}$$

$$L1 := 2 \quad L2 := 2 \quad r1 := \begin{pmatrix} 0-3 \\ 1-1 \\ 3-2 \end{pmatrix} \quad e1 := \frac{r1}{|r1|} \quad r2 := \begin{pmatrix} 0-3 \\ 1-1 \\ 1-2 \end{pmatrix}$$

$$e2 := \frac{r2}{|r2|} \quad r3 := \begin{pmatrix} 0-3 \\ L1-1 \\ L2-2 \end{pmatrix} \quad e3 := \frac{r3}{|r3|} \quad rf = \begin{pmatrix} 4-3 \\ .9-1 \\ 2.1-2 \end{pmatrix}$$

$$ef := \frac{rf}{|rf|} \quad e1 = \begin{pmatrix} -0.949 \\ 0 \\ 0.316 \end{pmatrix} \quad e2 = \begin{pmatrix} -0.949 \\ 0 \\ -0.316 \end{pmatrix} \quad e3 = \begin{pmatrix} -0.949 \\ 0.316 \\ 0 \end{pmatrix}$$

$$ef = \begin{pmatrix} 0.99 \\ -0.099 \\ 0.099 \end{pmatrix}$$

$$m := \text{ augment } (e1, e2) \quad c := \text{ augment } (m, e3)$$

$$F := 5000 \cdot ef \quad F = \begin{pmatrix} 4950.74 \\ -495.07 \\ 495.07 \end{pmatrix} \quad \begin{pmatrix} T1 \\ T2 \\ T3 \end{pmatrix} := c^{-1} \cdot (-F)$$

$$T1 = 1043.71 \quad T2 = 2609.27 \quad T3 = 1565.56$$

b) The range variable function can be used to solve the design problem: $L2 := .1, .5..4$

$$L1 := 2 \quad r1 := \begin{pmatrix} 0-3 \\ 1-1 \\ 3-2 \end{pmatrix}$$

$$s(L2) := L2 - 2 \quad e1 := \frac{r1}{|r1|} \quad r2 := \begin{pmatrix} 0-3 \\ 1-1 \\ 1-1 \end{pmatrix} \quad e2 := \frac{r2}{|r2|}$$

$$r3(L2) := \begin{pmatrix} 0-3 \\ L1-1 \\ s(L2) \end{pmatrix} \quad rf := \begin{pmatrix} 4-3 \\ .9-1 \\ 2.1-2 \end{pmatrix} \quad ef := \frac{rf}{|rf|} \quad e3(L2) := \frac{r3(L2)}{|r3(L2)|}$$

$$m := \text{ augment } (e1, e2) \quad c(L2) := \text{ augment } (m, e3(L2))$$

$$F := 5000 \cdot ef \quad F = \begin{pmatrix} 4950.74 \\ -495.07 \\ 495.07 \end{pmatrix} \quad T(L2) := c(L2)^{-1} \cdot (-F)$$

Note that as the value of $L2$ increases, the support moves farther out on the block and the tension in each member changes until near 3.7 m the tension in cable 1 becomes negative, indicating compression which cannot be supported by a cable. The minimum values of tension in all three cables occurs near $L2 = 1$ m where all tensions are less than 200N. This can be confirmed by using the root function: $x := 0.9 \;\; \text{root}(T(x)_0 - T(x)_1, x) = 1$. Next an optional study is set up which allows both of the coordinates of the cable to vary.

$$i := 0..9 \quad j := 0..20 \quad L1_i := 0.1 + i \cdot 0.2 \quad L2_j := 0 + j \cdot 0.2$$

$$r1 := \begin{pmatrix} 0-3 \\ 1-1 \\ 3-2 \end{pmatrix}$$

$$s(L2) := L2 - 2 \quad e1 := \frac{r1}{|r1|} \quad r2 := \begin{pmatrix} 0-3 \\ 1-1 \\ 1-2 \end{pmatrix} \quad e2 := \frac{r2}{|r2|}$$

$$r3(L1, L2) := \begin{pmatrix} 0-3 \\ L1-1 \\ s(L2) \end{pmatrix} \quad rf := \begin{pmatrix} 4-3 \\ .9-1 \\ 2.1-2 \end{pmatrix} \quad ef := \frac{rf}{|rf|}$$

$$e3(L1, L2) = \frac{r3(L1, L2)}{|r3(L1, L2)|}$$

$$m := \text{ augment } (e1, e2) \quad c(L1, L2) := \text{ augment } (m, e3(L1, L2))$$

$$F := 5000 \cdot ef \quad F = \begin{pmatrix} 4950.74 \\ -495.07 \\ 495.07 \end{pmatrix} \quad T(L1, L2) := c(L1, L2)^{-1} \cdot (-F)$$

$$M_{(i,j)} := T(L1_i, L2_j)_2$$

■

3.22 A free body diagram of this system is given in figure S3.22.

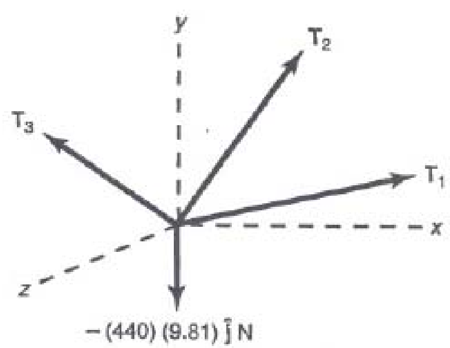

FIGURE S3.22

A unit vector along T_1, is

$$\hat{\mathbf{t}}_1 = \frac{1}{\sqrt{6^2+6^2}}(6\hat{\mathbf{i}}+6\hat{\mathbf{j}}) = 0.707\hat{\mathbf{i}} + 0.7070\hat{\mathbf{j}}.$$

A unit vector along T_2 is

$$\hat{\mathbf{t}}_2 = \frac{1}{\sqrt{6^2+5^2}}(6\hat{\mathbf{j}}-5\hat{\mathbf{k}}) = 0.768\hat{\mathbf{j}} - 0.640\hat{\mathbf{k}}.$$

Likewise

$$\hat{\mathbf{t}}_3 = \frac{1}{\sqrt{2^2+6^2+3^2}}(-2\hat{\mathbf{i}}+6\hat{\mathbf{j}}+3\hat{\mathbf{k}}) = -0.286\hat{\mathbf{i}} + 0.857\hat{\mathbf{j}} + 0.429\hat{\mathbf{k}}.$$

The forces $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$ can now be expressed in terms of 3 unknown tudes T_1, T_2, and T_3 as

$$\mathbf{T}_1 = T_1\hat{\mathbf{t}}_1,$$

$$\mathbf{T}_2 = T_2\hat{\mathbf{t}}$$

and

$$\mathbf{T}_3 = T_3\hat{\mathbf{t}}_3.$$

The equation of equilibrium is

$$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 - 4316\hat{\mathbf{j}} = 0.$$

In component form:

$$\begin{aligned} &.707T_1 + 0T_2 - .286T_3 = 0 \\ &.707T_1 + .768T_2 + .857T_3 = 4316 \text{ or} \\ &0 - .640T_2 + .429T_3 = 0 \end{aligned}$$

$$\begin{bmatrix} .707 & 0 & -.286 \\ .707 & .768 & .857 \\ 0 & -.640 & .429 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4316 \\ 0 \end{bmatrix}$$

which has solution

$$\underline{\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} 1052 \\ 1744 \\ 2604 \end{pmatrix} \text{ N}}$$

or in vector form

$$\mathbf{T}_1 = 744\hat{\mathbf{i}} + 744\hat{\mathbf{j}}(N)$$

$$\mathbf{T}_2 = 1399\hat{\mathbf{j}} - 116\hat{\mathbf{k}}(N)$$

$$\mathbf{T}_3 = -744\hat{\mathbf{i}} + 2232\hat{\mathbf{j}} + 1116\hat{\mathbf{k}}(N)$$

3.28 First define a unit vector along the rod:

$$\mathbf{u} = \frac{1}{\sqrt{45}}(-4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) = -0.596\mathbf{i} - 0.298\mathbf{j} + 0.745\mathbf{k}.$$

The position vector pointing to A is

$$\mathbf{r_A} = 4\mathbf{i} + 4\mathbf{j} + 0.2\mathbf{u} = 3.881\mathbf{i} + 3.940\mathbf{j} + 0.149\mathbf{k}.$$

A vector pointing in the direction of the tension can be obtained from:

$$\mathbf{r_D} - \mathbf{r_A} = 3.119\mathbf{i} - 3.940\mathbf{j} + 7.851\mathbf{k}$$

$$\mathbf{t} = \frac{\mathbf{r_D} - \mathbf{r_A}}{|\mathbf{r_D} - \mathbf{r_A}|} = 0.335\mathbf{i} - 0.423\mathbf{j} + 0.842\mathbf{k}.$$

The component of the tension along the rod must counteract the component of the weight along the rod:

$$-\mathbf{W} \cdot \mathbf{u} = \mathbf{T} \cdot \mathbf{u}$$

$$87.743 = 0.554T$$

$$T = 158.381 \text{ N}$$

$$\mathbf{T} = 158.381\mathbf{t} = 53.0\mathbf{i} - 66.949\mathbf{j} + 133.392\mathbf{k} \text{ N}.$$

The normal force must counteract the component of the tension normal to the rod:

$$\mathbf{T}_{\parallel} = (\mathbf{T} \cdot \mathbf{u})\mathbf{u} = -52.344\mathbf{i} - 26.172\mathbf{j} + 65.430\mathbf{k} \text{ N}$$

$$\mathbf{N} = -\mathbf{T}_{\perp} = -(\mathbf{T} - \mathbf{T}_{\parallel}) = -105.342\mathbf{i} + 40.778\mathbf{j} - 67.963\mathbf{k}.$$

3.37 First calculate the normal to the plane:

$$\mathbf{AB} = 4\mathbf{i} - 20\mathbf{k}$$

$$\mathbf{AC} = 6\mathbf{j} - 20\mathbf{k}$$

$$\mathbf{n} = \frac{\mathbf{AB} \times \mathbf{AC}}{|\mathbf{AB} \times \mathbf{AC}|} = 0.821\mathbf{i} + 0.547\mathbf{j} + 0.164\mathbf{k}.$$

Define the weight vector:

$$\mathbf{W} = -160\mathbf{k} \text{ lb}$$

and take the dot product to get the normal component

$$\mathbf{W_n} = (\mathbf{W} \cdot \mathbf{n})\mathbf{n} = -21.56\mathbf{i} - 14.37\mathbf{j} - 4.31\mathbf{k} \text{ lb}.$$

Finally, the tangential force can be obtained from:

$$\mathbf{W_t} = -(\mathbf{W} - \mathbf{W_n}) = -21.56\mathbf{i} - 14.37\mathbf{j} + 155.69\mathbf{k} \text{ lb}.$$