7.6 a) The free body diagrams of each joint and of the entire body are first constructed assuming each member to be in tension. These are given in the figure.

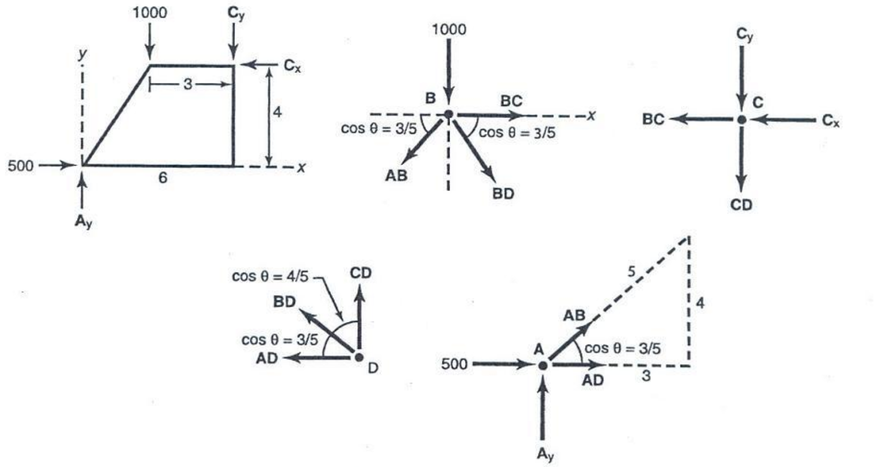

FIGURE S7.6

Note that the geometry is such that each angle is defined by a 3-4-5 triangle. The joint free body diagrams all contain more than two unknowns, so starting with the FBD's of the whole body yields

$$\sum M_c = 0 : (-6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) \times (500\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (-3\hat{\mathbf{i}}) \times (-1000\hat{\mathbf{j}}) = 0$$

or

$$-6A_y\hat{\mathbf{k}} + 2000\hat{\mathbf{k}} + 3000\hat{\mathbf{k}} = 0 \text{ or } \underline{A_y = 833 \text{ N (up)}}$$

$$\sum F_x = 0 : -C_x + 500 = 0 \text{ or } \underline{C_x = 500 \text{ (left)}}$$

$$\sum F_y = 0 : -C_y = 1000 + 833 = 0 \text{ or } \underline{C_y = -167 \text{ N (up)}}$$

Next consider the equilibrium equation for joint C

$$\sum F_x = 0 : -BC - C_x = 0 \text{ or } \underline{BC} = -C_x = \underline{-500 \text{ N (compression)}}$$

$$\sum F_y = 0 : -CD - C_y = 0 \text{ or} \underline{CD} = -C_y = \underline{167 \text{ N (tension)}}$$

The FBD for D yields

$$\sum F_x = 0 : -AB - BD3/5 = 0 \text{ or } AD = -3/5BD$$

$$\sum F_y = 0 : CD + BD4/5 = 0$$

or

$$\underline{BC} = 5/4(CD) = \underline{-209 \text{ N (compression) so that } AD = 125 \text{ N (tension)}}$$

The FBD for A yields

$$\sum F_x = 0 : 500{+}AD{+}AB(3/5) = 0 \text{ or } \underline{AB} = -5/3(625) = \underline{-1042 \text{ N (compression)}}$$

$\sum F_y$ can be used as a check and again yields $A_y = 833$ N. The FBD for B can be used as a check.

Alternately, the matrix approach can be used. In this case only the free body equations for joints A, B, C and D are needed.

From A:

$$(3/5)AB + AD + 500 = 0 \quad (x)$$
$$(4/5)AB + A_y = 0 \quad (y)$$

From B:

$$-(3/5)AB + (3/5)BD + BC = 0 \quad (x)$$
$$-(4/5)AB - (4/5)BD = 1000 \quad (y)$$

From C:

$$-C_x - BC = 0 \quad (x)$$
$$-C_y - CD = 0 \quad (y)$$

From D:

$$-AD - BD(3/5) = 0 \quad (x)$$
$$CD + BD(4/5) = 0 \quad (y)$$

or in matrix form:

$$\begin{bmatrix} 3/5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4/5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -3/5 & 0 & 1 & 3.5 & 0 & 0 & 0 & 0 \\ -4/5 & 0 & 0 & -4/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & -3/5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/5 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} AB \\ AD \\ BC \\ BD \\ CD \\ A_y \\ C_x \\ C_y \end{bmatrix} = \begin{bmatrix} -500 \\ 0 \\ 0 \\ 1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving via Mathcad matrix inversion yields

$$\begin{array}{ll} AB = -1042 \text{ N (compression)} & A_y = 833 \text{ N (up)} \\ AD = 125 \text{ N (tension)} & C_x = 500 \text{ N (left)} \\ BC = -500 \text{ N (compression)} & C_y = -166.7 \text{ N (up)} \\ BD = -208 \text{ N (compression)} & \\ CD = 166.7 \text{ N (tension)} & \end{array}$$

b) This is exactly the same as part a except that the y equation for joint D becomes $CD + (4/5)BD - 100 = 0$. Thus the matrix equation used in part A can be copied and the last element in the load vector changed from 0 to 100. The new solution is

$$\begin{array}{ll} AB = -1042 \text{ N (compression)} & A_y = 833 \text{ N}(up) \\ AD = 125 \text{ N (tension)} & C_x = 500 \text{ N}(left) \\ BC = -500 \text{ N (compression)} & C_y = -267 \text{ N}(up) \\ BD = -208 \text{ N (compression)} & \\ CD = 267 \text{ N (tension)} & \end{array}$$

The only elements that changed are the tension in CD and the reaction C_y.

7.17 A free body diagram (S7.17) of each pin followed by an equilibrium analysis yields (8 equations in 8 unknowns). From the geometry $\theta = \tan\frac{.5}{1} = 26.56°$. From A:

$$\sum F_x = 0 : -A_x + AC\cos 26.56° = 0 \text{ or } -A_x + 0.8944AC = 0 \qquad (1)$$

$$\sum F_y = 0 : \; A_y - AB - AC\sin 26.56° = 0 \text{ or } A_y - AB - 0.4472AC = 0 \qquad (2)$$

From B:

$$\sum F_x = 0 : B_x + 0.8944BC + BD = 0 \qquad (3)$$

$$\sum F_y = 0 = AB + .4472BC - 122.6 = 0 \qquad (4)$$

From C:

$$\sum F_x = 0 : -0.8994AC - 0.8994BC + 0.8994CD = 0 \qquad (5)$$

$$\sum F_y = 0 : .4427AC - .4427BC - .4427CD = 0 \qquad (6)$$

From D:

$$\sum F_x = 0 : -CD(0.8944) - BD = 0 \qquad (7)$$

$$\sum F_y = 0 : 0.4472CD - 122.6 = 0 \qquad (8)$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0.8944 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -0.4472 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0.8944 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0.4472 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.8994 & -0.8994 & 0 & 0.8994 \\ 0 & 0 & 0 & 0 & .4472 & -.4472 & 0 & -.4472 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -0.8944 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .4472 \end{bmatrix} \begin{pmatrix} A_x \\ A_y \\ B_x \\ AB \\ AC \\ BC \\ BD \\ CD \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 122.6 \\ 0 \\ 0 \\ 0 \\ 122.6 \end{pmatrix}$$

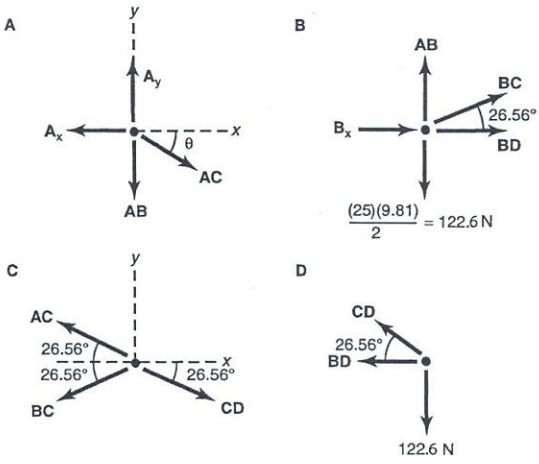

FIGURE S7.17

Which has solution (in N)

$$\begin{array}{lll} A_\lambda = 245.2 & AB = 122.6 & BD = -240.3 \\ A_y = 245.2 & AC = 274.3 & CD = 274.3 \\ B_x = 245.2 & BC = 0 & \end{array}$$

7.23 A free-body diagram of each joint (S7.23) and the corresponding equations of equilibrium are:

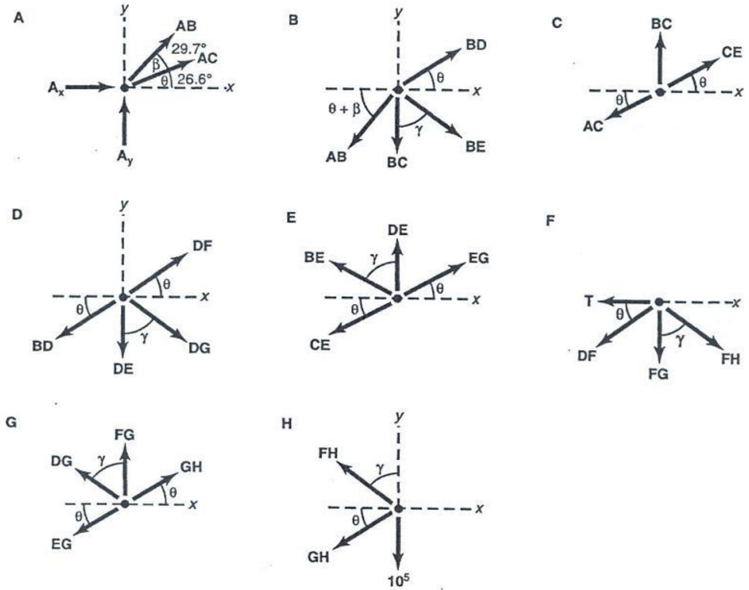

FIGURE S7.23

A:

$$\sum F_x = 0 : A_x + AB\cos(\theta+\beta) + AC\cos\theta = 0 \qquad (1)$$

$$\sum F_y = 0 : A_y + AB\sin(\theta+\beta) + AC\sin\theta = 0 \qquad (2)$$

B:

$$\sum F_x = 0 :$$

$$-AB\cos(\theta+\beta) + BE\sin\gamma + BD\cos\theta = 0 \qquad (3)$$

$$\sum F_y = 0 :$$

$$-AB\sin(\theta+\beta) - BC + BD\sin\theta - BE\cos\gamma = 0 \qquad (4)$$

C:

$$\sum F_x = 0 : -AC\cos\theta + CE\cos\theta = 0 \qquad (5)$$

$$\sum F_y = 0 : -AC\sin\theta + BC + CE\sin\theta = 0 \qquad (6)$$

(students will note that $BC = 0$ and $CE = AC$)

D:

$$\sum F_x = 0 : -BD\cos\theta + DF\cos\theta + DG\sin\gamma = 0 \qquad (7)$$

$$\sum F_y = 0 :$$

$$-BD\sin\theta - DE + DF\sin\theta - DG\cos\gamma = 0 \qquad (8)$$

E:

$$\sum F_x = 0 : -BE\sin\gamma - CE\cos\theta + EG\cos\theta = 0 \qquad (9)$$

$$\sum F_y = 0 :$$

$$BE\cos\gamma - CE\sin\theta + DE + EG\sin\theta = 0 \qquad (10)$$

F: (treating the double cable as "one")

$$\sum F_x = 0 : -T - DF\cos\theta + FH\sin\gamma = 0 \qquad (11)$$

$$\sum F_y = 0 : -DF\sin\theta - FG - FH\cos\gamma = 0 \qquad (12)$$

G:

$\sum F_x = 0:$

$$-DG\sin\gamma - EG\cos\theta + GH\cos\theta = 0 \tag{13}$$

$\sum F_y = 0:$

$$DG\cos\gamma - EG\sin\theta + FG + GH\sin\theta = 0 \tag{14}$$

H: (treating the double cable as "one")

$$\sum F_x = 0: -FH\sin\gamma - GH\cos\theta = 0 \tag{15}$$

$$\sum F_y = 0: FH\cos\gamma - GH\sin\theta - 10^5 = 0 \tag{16}$$

The solution is

$\underline{A_x = 21,960 \text{ lb}}$ $\underline{AB = 1799 \text{ lb}}$ $\underline{BE = 1192 \text{ lb } (C)}$

$\underline{A_y = 10,000 \text{ lb}}$ $\underline{AC = -25680 \text{ lb } (C)}$ $\underline{BD = 2334 \text{ lb}}$

$\underline{T = 7890 \text{ lb}}$ $\underline{BC = 0}$ $\underline{CE = -25,680 \text{ lb } (C)}$

$\underline{DE = 997 \text{ lb}}$ $\underline{EG = -26910 \text{ lb } (C)}$ $\underline{GH = -12420 \text{ lb } (C)}$

$\underline{DF = 3592 \text{ lb}}$ $\underline{FH = 11,960 \text{ lb}}$

$\underline{DG = -1192 \text{ lb } (C)}$ $\underline{GH = -6049 \text{ lb } (C)}$